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# Endogenous Movement and Equilibrium Selection in Spatial Coordination Games

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**Abstract** We study the effects of agent movement on equilibrium selection in network based spatial coordination games with Pareto dominant and risk dominant Nash equilibria. Our primary interest is in understanding how endogenous partner selection on networks influences equilibrium selection in games with multiple equilibria. We use agent based models and best response behaviors of agents to study our questions of interest. In general, we find that allowing agents to move and choose new game play partners greatly increases the probability of attaining the Pareto dominant Nash equilibrium in coordination games. We also find that agent diversity increases the ability of agents to attain larger payoffs on average.

**Keywords** Coordination games · Movement · Equilibrium selection · Agent based modeling

## 1 Introduction

There exists a vast literature on the emergence of cooperation and altruistic behavior both in real world situations and in theoretical games. Much of this literature centers around prisoner's dilemma games either directly or as a metaphor for a real world scenario. A related literature studies the results of coordination games where agents receive payoffs if they are able to coordinate on the same strategy but pay a cost if they

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do not. For example, it is beneficial to an agent if her friends own computers that use the same operating system as she does. However, it may be inconvenient and costly if they do not (e.g. sharing files may be difficult). Continuing with this example, it may be that one of the potential operating system choices is better than the others. If agents coordinate on this strategy, everyone benefits. However, a tension may exist if the better operating system also has a steeper learning curve. Thus, if an agent chooses the better operating system, but no one else does, it may be very costly to her because she will not have friends to consult when problems occur.

To translate this situation to a formal game, consider the following generic two agent, two strategy simultaneous game:

| | | Player 2 | |
|---|---|---|---|
| | | X | Y |
| Player 1 | X | $a, a$ | $b, c$ |
| | Y | $c, b$ | $d, d$ |

Throughout the paper we assume $a > c,\ d > b$ such that there exist two pure strategy Nash equilibria, $X, X$ and $Y, Y$. Agents attempt to coordinate with their game play partners on one of the two Nash equilibria. Thus, our game of interest is a standard $2 \times 2$ coordination game. Further, we assume that $a > d$, such that $X, X$ is the Pareto dominant Nash equilibrium. Harsanyi and Selten (1988) define equilibrium $Y, Y$ to be a risk dominant Nash equilibrium if $(a - c)(a - c) < (d - b)(d - b)$ which is equivalent to $a + b < c + d$. Our primary interest in this paper will be with payoffs assigned such that $Y, Y$ qualifies as risk dominant. We study equilibrium selection in these games using an agent-based model.

As in the operating system example, note that there is a tension between agents attempting to coordinate on the Pareto dominant versus the risk dominant Nash equilibrium. All agents would prefer to coordinate on $X, X$, because each agent receives a larger payoff than in $Y, Y$. But, should coordination not occur (one agent playing X and the other playing Y), the agent playing $X$ is penalized with a low payoff of $b$. More importantly, as $b$ decreases, playing X becomes more risky and playing Y becomes more attractive.

There exists a large literature on the long run selection of equilibria in these coordination games (without agent movement). As examples, Ellison (1993); Kandori et al. (1993) and Young (1993) study equilibrium selection in an evolutionary framework where agents are randomly matched with game partners. They find that the risk dominant Nash equilibrium is the unique stochastically stable equilibrium when agents have a small probability of making mistakes in strategy selection. Morris (2000) studies the spread of a Pareto dominant Nash equilibrium where agents play a spatial coordination game on various topologies. He finds that a Pareto dominant equilibrium may be favored in some network based coordination games if the number of neighbors in the network expands at an intermediate rate (quickly, but not too quickly).

In this paper, we explore how endogenous agent movement (which has not been previously studied in coordination games) affects the equilibrium selection results described above. Specifically, agents are located on a two-dimensional lattice. Agents play a coordination game with each nearest neighbor on the network in each period.

Agents choose a best response to last period's play by their neighbors as their action in the current period. Using agent based modeling, we study the evolution of the agent strategies and the attainment of Pareto versus risk dominant Nash equilibria.

Agent movement has previously been studied in spatial prisoner's dilemma games. In this field, researchers are interested in whether the ability of agents to move favors the invasion of defecting agents into neighborhoods of cooperators, or whether the ability of agents to move allows cooperators to escape defectors. Previous research suggests that the ability of agents to move enhances rates of cooperation on average. For example, Aktipis (2004) studies the behavior of a "walk-away" strategy in a spatial prisoner's dilemma game where agents cooperate if a rival cooperated in a previous period or move to a new location if the rival defected in the previous period. She finds that walk-away is a successful strategy when placed in an Axelrod (1984) style tournament among commonly studied strategies such as tit-for-tat. Helbing and Yu study the emergence of cooperation in a spatial prisoner's dilemma where agents both imitate neighbors with high paying strategies and move to nearby locations that yield higher payoffs. They term this strategy "success driven migration" and find it to robustly lead to cooperative outcomes in a variety of situations and noise. Barr and Tassier (2010) study the rates of cooperation and evolution of mixed strategies in a spatial prisoner's dilemma game where agents are allowed to move. They find that the opportunity to move greatly enhances the probability of agent cooperation across many network structures. Another related literature studies the ability of agents to maintain or sever interactions in prisoner's dilemma games on evolving heterogeneous networks (Ashlock et al. 1996; Biely et al. 2007; Hanaki et al. 2007; Santos et al. 2006; Van Segbroeck et al. 2008). While fundamentally different, both the movement mechanism studied in our paper and in the literature mentioned above and a severing ties/evolving networks approach both lead to the ability of cooperating agents to avoid defecting agents in a prisoner's dilemma.

Echoing the benefits of movement in prisoner's dilemma games, we find that adding agent movement into a coordination game also increases the likelihood of good outcomes. Specifically, we find that the Pareto dominant Nash equilibrium is attained much more frequently when agents have the ability to move on the network and choose game play partners, than when agents are not allowed to move. We also study the effects of diverse agent types. We find that diversity in payoffs of the agent types allows for strategies to survive longer in the populations and promotes the attainment of higher payoffs on average.

## 2 Our Model

Each run of our model proceeds as follows. In the initialization procedure, N agents are created and assigned a random location on an $L \times L$ lattice with fixed boundaries. Note that we set $N$ to be less than $L \times L$ so that there are some vacant locations. In addition, each agent is assigned a random strategy, X or Y, according to a specified probability distribution. This random initial strategy is played in the first period of the model.

Following the initialization procedure, each agent plays the coordination game described in the previous section, with each nearest neighbor on the lattice.[1] In each period, each agent chooses a single action that must be played with every neighbor. Agents choose this action as follows: The action in period one is assigned by the initialization procedure. In each subsequent period, agents choose an action that is a best response to their neighbors' actions in the previous period. Specifically, an agent calculates her average payoff for both strategy X and strategy Y against the strategies chosen by each of her neighbors in the previous period. Whichever strategy yields a larger average payoff is chosen in the current period. Ties are broken by the agent playing the strategy that she most recently played.[2]

Following agent game play, each agent is individually given an opportunity to move to a new vacant location on the lattice with probability $m$. If the agent is given this opportunity, a random vacant location is chosen from among the set of vacant locations on the lattice with uniform probability. The agent then calculates the best response strategy at the new location, X or Y, and the corresponding average payoff of that strategy. The agent then compares the average payoff of the best response at the new location to the average payoff of the best response at the current location. If this average payoff is greater at the new location, she moves there. Otherwise, she remains at her current location. We then repeat this game play procedure until we generate equilibrium behavior. We repeat the entire process (initialization and game play) for R runs for a specified set of parameters. We take averages over these R runs and report results for each parameter set below.

## 3 Results

We report average results below for R = 5,000 runs of each parameter set. We are primarily interested in equilibrium selection differences when movement is allowed in the model versus when movement is not allowed. Therefore, we vary the probability of a movement opportunity between m = 0 and m = 1 across different sets of runs and compare the equilibrium selection results. Unless otherwise noted, all experiments consist of N = 100 agents and payoffs of $a = 2.0$, $c = 0.0$, and $d = 1.0$.

### 3.1 Variation in Risk

We begin this comparison with the following payoff selections: $a = 2.0,\ c = 0.0,\ d = 1.0$ so that we have the following normal form game representation:

[1] In this set-up an agent can have a maximum of eight neighbors but some agents may have fewer because of vacant locations or boundaries.

[2] One motivation for this assumption is that agents must pay some small (relative to the payoffs) positive cost to switch strategies. However, ties are exceptionally rare and for some payoff matrices are not possible. Thus this tie-breaking assumption does not play a meaningful role in our results.

| | Player 2 | |
|---|---|---|
| Player 1 | X | Y |
| X | 2, 2 | $b$, 0 |
| Y | 0, $b$ | 1, 1 |

We then vary b at intervals between 0 and −6. Note that $X, X$ is the Pareto dominant Nash equilibrium. Also, note that $b \leq 1$ implies that $Y, Y$ is a Nash equilibrium and $b < -1$ implies that $Y, Y$ is a risk dominant Nash equilibrium. Initially, we set $N = 100$ and $L = 12$, so that there are 144 locations: 100 locations with agents and 44 vacant locations. For these runs we also set the probability of playing X in the first period equal to 50 %. Thus, on average, there will be 50 % of agents playing X in period one and 50 % playing Y in period one.

In Table 1 we report the average percent of agents that play strategy X in equilibrium when no movement is allowed, $m = 0$, and when movement is allowed for each agent in every period, $m = 1$, as a function of the payoff parameter $b$.

To begin, note the behavior of the model when no movement is allowed. As expected, the percent of agents playing the Pareto dominant Nash equilibrium strategy, X, decreases as the payoff $b$ decreases (as $b$ decreases, playing X becomes more risky). Further, recall that when $b < -1$, Y, Y becomes a risk dominant Nash equilibrium. And, we see in the table that for $b > -1$ a majority of agents play the Pareto dominant Nash equilibrium strategy. But, for $b < -1$, a majority of agents play strategy Y that corresponds to the risk dominant Nash equilibrium. At $b = -1$ we observe an even

**Table 1** Percent of agents that coordinate on the Pareto dominant strategy, X, as a function of the payoff b

| b | m = 0 | | | m = 1 | | |
|---|---|---|---|---|---|---|
| | Mean (%) | SD (%) | Time | Mean (%) | SD (%) | Time |
| 0 | 98.3 | 3.4 | 1.0 | 100.0 | 0.0 | 4.0 |
| −0.5 | 80.2 | 13.3 | 1.8 | 100.0 | 0.0 | 5.9 |
| −1 | 50.2 | 12.8 | 2.6 | 99.9 | 2.4 | 11.9 |
| −1.1 | 24.2 | 14.5 | 4.3 | 99.9 | 2.4 | 12.4 |
| −1.5 | 24.3 | 14.7 | 4.3 | 99.0 | 8.7 | 19.4 |
| −2 | 9.1 | 9.1 | 4.4 | 72.2 | 38.4 | 70.6 |
| −2.5 | 2.8 | 4.8 | 4.0 | 51.1 | 42.8 | 82.0 |
| −3 | 1.6 | 3.4 | 3.4 | 14.7 | 26.0 | 53.7 |
| −3.5 | 0.4 | 1.3 | 2.7 | 8.2 | 18.5 | 34.3 |
| −4 | 0.3 | 1.1 | 2.6 | 5.0 | 13.3 | 22.5 |
| −4.5 | 0.3 | 1.1 | 2.5 | 4.4 | 11.7 | 23.1 |
| −5 | 0.3 | 1.0 | 2.4 | 1.9 | 6.6 | 14.0 |
| −5.5 | 0.1 | 0.5 | 2.1 | 0.6 | 3.2 | 6.3 |
| −6 | 0.1 | 0.5 | 2.1 | 0.7 | 2.9 | 6.6 |

Other payoffs: a = 2.0, c = 0.0, d = 1.0. No movement, m = 0 versus movement, m = 1. 12 × 12 lattice, $N = 100$ agents. Also reported are the average number of time periods taken to reach equilibrium and the SD split between the two strategies in equilibrium. These basic results correspond well with the existing literature on equilibrium selection in coordination games reported in the introduction. Although our model is different (network based matching vs random matching), the risk dominant equilibrium is still favored in our model when movement is not allowed.

More interesting is the comparison of the results when movement is not allowed to the results when movement is allowed. As one can see in the table, allowing movement greatly increases the probability that the Pareto dominant strategy is played in equilibrium. Even at levels where costs of non-coordination are fairly large such as $b = -2.5$ the Pareto dominant Nash equilibrium is still played in a majority of runs (51.1 %). Without movement, only 2.8 % of runs result in the Pareto dominant Nash equilibrium at $b = -2.5$. Further, when movement is allowed, it is still possible to generate small numbers of agents who play the Pareto dominant strategy in equilibrium even when doing so is very risky, when $b$ is very small.

The ability of agents to more frequently coordinate on the Pareto dominant Nash equilibrium appears to primarily depend on two things: the ability of agents to form pockets of agents playing the $X, X$ Nash equilibrium early in the run and the ability of strategy $X$ to survive until these pockets can form. As intuition, imagine a small pocket (maybe only two or three agents) playing $X, X$ with each other and with no strategy $Y$ agents connected to them. If there is sufficient room around this small group of agents, then more agents can join the group; the $X, X$ group grows. However if the network is very dense, it may be unlikely that the small group can form in isolation away from agents playing $Y$. In this case it is difficult for the $X, X$ strategy to spread because new agents joining the group may have to play with another agent playing $Y$ nearby. If $b$ is small enough it may not be optimal for the new agent to play $X$ when one of his games will be against an agent playing $Y$. The $X, X$ group cannot grow. Further, if the new agent plays $Y$, it may tip some of the $X, X$ agents to playing $Y$. In this case, the $X, X$ group may shrink. With a less dense network it is less likely that an initial $X, X$ group will shrink and more likely that it will grow.[3] Second, and more simply, in order for the Pareto dominant Nash equilibrium to be attained, strategy $X$ must survive long enough in the population for agents to coordinate on it. We will discuss this further in Sect. 4 later in the paper.

Also note in the table the time to reach equilibrium. When movement is not allowed, the equilibrium is reached very quickly; less than five periods on average. When movement is allowed, it takes longer to reach an equilibrium. Partly, this occurs because the strategy space for the game is larger when movement is allowed. In addition, note that parameter values, such as $b = -2.5$, where each of the two equilibria result a significant number of times. Here it take much longer to reach equilibrium. As intuition suggests, if either equilibrium is likely to be reached, it may take more time periods for agents to coordinate on one of them. Thus even with movement, equilibrium is

[3] The effects of network density and clustering are also discussed in Nowak and May (1992) and Watts (1999).

**Table 2** Percent of agents that coordinate on the Pareto dominant strategy, X, as a function of population size, N

| Agents | m = 0 | | | m = 1 | | |
|---|---|---|---|---|---|---|
| | Mean (%) | SD (%) | Time | Mean (%) | SD (%) | Time |
| 70 | 19.0 | 9.3 | 3.1 | 69.9 | 2.4 | 13.1 |
| 85 | 20.9 | 11.5 | 3.6 | 84.9 | 3.1 | 14.4 |
| 100 | 24.2 | 14.6 | 4.4 | 99.0 | 8.8 | 19.8 |
| 115 | 24.3 | 18.0 | 5.2 | 96.4 | 34.7 | 39.8 |
| 120 | 25.7 | 19.6 | 5.6 | 82.9 | 43.1 | 44.6 |
| 130 | 22.0 | 21.4 | 6.5 | 40.5 | 37.7 | 50.8 |

Payoffs: a = 2.0, b = −1.5, c = 0.0, d = 1.0. No movement, m = 0 versus movement, m = 1. 12 × 12 lattice. Also reported are the average number of time periods taken to reach equilibrium and SD

reached fairly quickly when b is large or small, but for intermediate values, the time to reach equilibrium can be much longer as reported in the table.[4]

## 3.2 Variation in Population Size

Next we consider how the density of agents on the lattice changes equilibrium selection. As mentioned above, a more dense network should favor the risk dominant Nash equilibrium. Here we leave the payoffs for $a$, $c$, and $d$ as above and set $b = -1.5$ (an intermediate value for b where we had strong effects for movement, as can be seen in Table 1). Again we set the initial strategy distribution equal to 50 % for each strategy.

As reported in Table 2, changing the agent density does little to the equilibrium selection results when movement is not allowed. A more dense network slightly decreases the probability of coordinating on the Pareto dominant Nash equilibrium. However, when movement is allowed, a more dense lattice makes it more difficult to coordinate on the Pareto dominant Nash equilibrium, as expected. Again this occurs for two reasons: First, when the network is very dense, it may be more difficult to find a vacant location near a group of $X$, $X$ coordinating agents. Second, because the network is more dense it is difficult for $X$, $X$ coordination to spread. Each location will have more occupied locations adjacent to it. Thus it may be difficult for a small group of agents to "tip" toward the Pareto dominant Nash equilibrium. When the network is less densely populated it may be easy to find small groups of unconnected agents that can coordinate on the $X$, $X$ Nash equilibrium. Then once these agents coordinate, movement into adjacent vacant cells can allow this equilibrium to spread. This process is more difficult when the lattice is densely populated.

[4] We consider two robustness tests of our base results in the Appendix to this paper: random movement and a torus based model that eliminates edge effects.

**Table 3** Percent of agents that coordinate on the Pareto dominant strategy, X, as a function of the initial percentage of strategy X agents

| P(x) | m = 0 | | | m = 1 | | |
|---|---|---|---|---|---|---|
| | Mean (%) | SD (%) | Time | Mean (%) | SD (%) | Time |
| 70 | 76.9 | 12.5 | 2.7 | 100.0 | 0.0 | 4.2 |
| 60 | 51.1 | 16.7 | 3.8 | 100.0 | 0.0 | 7.7 |
| 50 | 24.4 | 14.6 | 4.3 | 98.9 | 9.5 | 19.5 |
| 40 | 8.5 | 8.7 | 3.7 | 83.7 | 34.4 | 42.6 |
| 30 | 2.5 | 4.4 | 2.7 | 47.5 | 47.1 | 41.3 |
| 20 | 0.5 | 1.7 | 1.9 | 15.7 | 34.5 | 18.6 |

Payoffs: a = 2.0, b = −1.5, a = 0.0, d = 1.0. No movement, m = 0 versus movement, a = 1. 12 × 12 lattice, $N = 100$ agents. Also reported are the average number of time periods taken to reach equilibrium and SD

## 3.3 Variation in Initial Strategy

Next, we vary the initial strategy distribution in the population. Again we leave the payoffs unchanged with $a = 2.0$, $b = -1.5$, $c = 0.0$, and $d = 1.0$ and return to $N = 100$ agents. But, we vary the initial percentage of agents playing strategy X in period one from 20 to 70 % (with the complement playing Y). We report the results in Table 3.

There are two items of note in the table. First, the initial distribution of agents has a large effect on the equilibrium selected. Moving the initial percentage of agents playing X slightly above (below) 50 % greatly increases (decreases) the probability of agents coordinating on the X, X Nash equilibrium. Moving the initial players of *X* above or below the 50 % threshold changes the typical majority of game play partner strategies to *X* or *Y*. Because of the positive feedbacks associated with best response dynamics in coordination games this tips the dynamic process of equilibrium selection toward the more common strategy. Of course, this dynamic process is affected by the size of the *b* payoff as well as the ability to move. If the *b* payoff is exceptionally small it will still be difficult to obtain the Pareto dominant Nash equilibrium. Second, movement can act with large force to counteract the initial distribution. For instance, when only 40 % of agents play strategy X in the initial period, only 8.5 % of agents play X in equilibrium when movement is not allowed. But, allowing movement increases the equilibrium incidence of X to 83.7 %. Even when only 30 % of agents play strategy X initially, movement can allow a significant percentage of agents to coordinate on the Pareto efficient strategy. In this case 47.5 % of agents play X in equilibrium. As described in the Sect. 3 of the paper, movement allows clusters of *X* playing agents to more easily form which then leads to neighbors being more and more likely to play *X*. Thus, as stated above, the ability to move and select game play partners can have a large effect on the equilibrium selected and may lead to a majority playing the Pareto dominant Nash equilibrium even when starting with fewer than 50 % of agents playing *X*. Most importantly we again see that movement greatly increases the likelihood of the population selecting the Pareto dominant Nash equilibrium.

## 4 Heterogeneous Agents

So far, all agents in our model have been identical in terms of payoffs. With these identical agents we have demonstrated that movement leads to much higher levels of coordination on the Pareto dominant Nash equilibrium. The effect of heterogeneous payoffs has been studied in other contexts. For instance, Fort (2008) examines the evolution of game payoffs and the emergence of cooperation in the resulting steady state. Bednar and Page (2007) study the evolution of strategies when heterogeneous agents have cognitive constraints and play an ensemble of heterogeneous games. Related to these studies we now consider agents of various types interacting together. Different types of agents have different payoffs in the same game. This may occur because agents differ in their risk preferences, or because they have different preferences over game outcomes. First we assume that one type of agent pays a larger cost from a lack of coordination when playing strategy X. Specifically, there are two types of agents Q and R. Payoffs for the game with heterogeneous agent types are shown below:

| | | Player 2 (type R) | |
|---|---|---|---|
| | | X | Y |
| Player 1 (type Q) | X | $a, a$ | $b_q, c$ |
| | Y | $c, b_r$ | $d, d$ |

The subscript on the $b$ payoff indicates the payoff to the agent of a particular type. We assume that $b_q > b_r$ so that attempting to coordinate on the Pareto dominant Nash equilibrium is more risky for a type R player. As an example, the payoffs for a game where a type 1 and a type 2 player interact may be $b_r = -4$ and $b_q = -1$ resulting in the following game:

| | | Player 2 (type R) | |
|---|---|---|---|
| | | X | Y |
| Player 1 (type Q) | X | 2, 2 | −1, 0 |
| | Y | 0, −4 | 1, 1 |

Of course, two players of the same type may interact and then the payoffs look as in the examples in the previous section. For two type Q agents this would be:

| | | Player 2 (type Q) | |
|---|---|---|---|
| | | X | Y |
| Player 1 (type Q) | X | 2, 2 | −1, 0 |
| | Y | 0, −1 | 1, 1 |

And, for two type R agents this would be:

| | | Player 2 (type R) | |
|---|---|---|---|
| | | X | Y |
| Player 1 (type R) | X | 2, 2 | −4, 0 |
| | Y | 0, −4 | 1, 1 |

Note that the type R agent still faces more risk for playing X regardless of what type agent she is paired with.

In this setup we want to investigate how the distribution of payoffs across the agent types may change the attainment of equilibria. To do so we hold the average payoff across agents in the population constant. We begin with 50 type Q and 50 type R agents in the population. We want to compare the effect of heterogeneous payoffs to a base case with homogeneous payoffs. As an example, set the base case payoffs as $b_q = b_r = -4$. We then vary $b_q$ and $b_r$ so that the average remains $-4$, (for example $b_q = -2$, $b_r = -6$). Thus the *average* agent in the population is constant (even though the average agent does not exist in the population in the second case). We report two sets of experiments under the conditions reported above in Tables 4 and 5.

As one can see in the tables, increasing the diversity of agent types typically has an effect that leads to a greater likelihood of coordinating on the Pareto dominant Nash equilibrium for both the no movement and movement scenarios. It appears that, once the system has some agents coordinating on the Pareto dominant equilibrium, that this group provides a seed from which further Pareto coordination occurs. The results are

**Table 4** Percent of agents that coordinate on the Pareto dominant strategy, X, as a function of $b_q$ and $b_r$ payoffs

| $b_q$ | $b_r$ | m = 0 | | | m = 1 | | |
|---|---|---|---|---|---|---|---|
| | | Mean (%) | SD (%) | Time | Mean (%) | SD (%) | Time |
| 0 | −8 | 28.0 | 8.4 | 3.3 | 63.1 | 23.2 | 52.4 |
| −1 | −7 | 5.9 | 6.1 | 3.7 | 15.4 | 16.3 | 32.5 |
| −2 | −6 | 1.0 | 2.5 | 3.1 | 7.2 | 12.8 | 27.5 |
| −3 | −5 | 0.6 | 1.8 | 2.8 | 4.6 | 11.4 | 24.6 |
| −4 | −4 | 0.4 | 1.4 | 2.6 | 4.7 | 12.2 | 21.2 |

Payoffs: a = 2.0, c = 0.0, d = 1.0. No movement, m = 0 versus movement, m = 1. 12 × 12 lattice, $N = 100$ agents. Also reported are the average number of time periods taken to reach equilibrium and SD

**Table 5** Percent of agents that coordinate on the Pareto dominant strategy, X, as a function of $b_q$ and $b_r$ payoffs

| $b_q$ | $b_r$ | m = 0 | | | m = 1 | | |
|---|---|---|---|---|---|---|---|
| | | Mean (%) | SD (%) | Time | Mean (%) | SD (%) | Time |
| 1 | −5 | 63.1 | 5.5 | 2.0 | 100.0 | 1.3 | 17.3 |
| 0.5 | −4.5 | 59.8 | 8.1 | 2.4 | 99.9 | 1.5 | 15.3 |
| 0 | −4 | 41.3 | 10.8 | 3.0 | 99.4 | 4.9 | 22.1 |
| −0.5 | −3.5 | 21.4 | 11.4 | 3.9 | 95.3 | 16.8 | 34.7 |
| −1 | −3 | 17.4 | 10.9 | 4.0 | 81.4 | 33.4 | 69.5 |
| −1.5 | −2.5 | 9.2 | 9.6 | 4.6 | 88.6 | 28.5 | 54.7 |
| −2 | −2 | 9.2 | 9.0 | 4.4 | 72.2 | 38.3 | 73.0 |

Payoffs: a = 2.0, c = 0.0, d = 1.0. No movement, m = 0 versus movement, m = 1. 12 × 12 lattice. Also reported are the average number of time periods taken to reach equilibrium and SD

**Table 6** Percent of agents that coordinate on the Pareto dominant strategy, X, as a function of the percent of type 1 (less risk) versus type 2 (more risk) agents

| % Type distribution | $b_q$ | $b_r$ | m = 0 | | | m = 1 | | |
|---|---|---|---|---|---|---|---|---|
| | | | Mean (%) | SD (%) | Time | Mean (%) | SD (%) | Time |
| Baseline (50 % each type) | −4 | −4 | 0.3 | 1.3 | 2.6 | 5.1 | 13.4 | 22.5 |
| Few type Q (20 %) | −2 | −5 | 0.7 | 1.9 | 2.8 | 6.7 | 12.7 | 28.4 |
| Many type Q (80 %) | −2 | −12 | 3.0 | 4.6 | 3.6 | 23.9 | 28.9 | 63.0 |

Payoffs: a = 2.0, c = 0.0, d = 1.0. No movement, m = 0 versus movement, m = 1. 12 × 12 lattice. Also reported are the average number of time periods taken to reach equilibrium and SD

strongest when one of the players faces no additional cost of trying to coordinate on the Pareto dominant Nash equilibrium (when $b_q$ is largest). Interestingly, we do not find evidence of sorting by type among the agents. The agents remain evenly mixed throughout all of these runs, suggesting that coordination on an equilibrium is fast relative to the spatial sorting of agents.

As a final example of agent heterogeneity we allow the share of agents who face a larger cost of non-coordination to vary but alter the $b_q$ and $b_r$ payoffs so that, again, the average agent stays the same. Again, we start with a base case where $b_q = b_r = -4$. Then we consider two variations: In the first experiment, we set $b_q = -2$ for 20 % of the agents. Then, in order to keep the average payoffs constant, we set $b_r = -4.5$ for the remaining 80 % of the agents (so that the average b payoff is still −4). In a second experiment, we again set $b_q = -2$ but we do so for 80 % of the agents. The remaining 20 % have $b_r = -12$ (again to keep the average b payoff at −4). One can view this experiment as thinking about policy implications for promoting the Pareto dominant outcome. Perhaps a government is able to provide assurances about payoffs in the event of non-coordination to the type q agents but in doing so places a larger cost onto the type r agents that fail to coordinate. The results are shown in Table 6.

We find that shifting a small share of the agents toward facing smaller non-coordination costs does little to move the system toward the Pareto dominant equilibrium. Instead shifting a large percentage toward smaller non-coordination costs and a small percentage toward very large non-coordination costs ($b_r = -12$) greatly enhances the ability of agents to coordinate on the Pareto dominant equilibrium when movement is allowed. Further, the results suggest that some agents facing large non-coordination costs should not impact equilibrium attainment in coordination games. And, it may be beneficial if the additional cost this small fraction face is distributed away from a large fraction of the population. Again, we do not find evidence of agent sorting by type. Equilibrium convergence appears to occur prior to any agent sorting.

## 4.1 Conflicting Preferences

As a final experiment, we consider a case where agent preferences may not be aligned concerning the Pareto dominant equilibrium. Again let there be two types of agents. This time label them as type E and type F. A type E agent faces payoffs that are identical in ordering to what has been discussed above. So that, if two type E agents are matched the game and payoffs are identical to the games already displayed.

| | | Player 2 (type E) | |
|---|---|---|---|
| | | X | Y |
| Player 1 (type E) | X | $a, a$ | $b, c$ |
| | Y | $c, b$ | $d, d$ |

Type F agents have reversed preference orderings. For a type F agent, coordination on Y, Y results in the large payoff of $a$ and the greatest cost of non-coordination occurs with strategy Y as well. Finally, for a type F agent, coordination on X, X results in a lower payoff of $d$. Thus if two type F agents are matched, the game payoff orderings are reversed so that the game appears as:

| | | Player 2 (type F) | |
|---|---|---|---|
| | | X | Y |
| Player 1 (type F) | X | $d, d$ | $c, b$ |
| | Y | $b, c$ | $a, a$ |

Here the agents prefer to coordinate on the bottom right Y,Y Pareto dominant Nash equilibrium (because $a > d$). Finally, if a type E agent is matched with a type F agent they play a game with the following payoff structure:

| | | Player 2 (type F) | |
|---|---|---|---|
| | | X | Y |
| Player 1 (type E) | X | $a, d$ | $b, b$ |
| | Y | $c, c$ | $d, a$ |

where again $a > c$ and $d > b$ so that there are two Nash equilibria, and we also continue to assume that $a > d$ so that the row player prefers the equilibrium corresponding to X,X and the column player prefers the equilibrium Y,Y. This is an example of what is commonly called the "battle of the sexes" game. As a more concrete example consider:

| | | Player 2 (type F) | |
|---|---|---|---|
| | | X | Y |
| Player 1 (type E) | X | 2, 1 | −2, −2 |
| | Y | 0, 0 | 1, 2 |

The issue here is obviously that the agents want to coordinate on one of the two Nash equilibria but the order of preference among the equilibria and the cost of non-coordination faced is reversed if playing an agent of the other type. Playing safe for one type is playing risky for the other type. Again, note that play between agents can be with an agent's own type or with the opposite type. And, the players cannot recognize the type of their opponents.

Again, we use the standard payoffs listed above $a = 2$, $c = 0$, and $d = 1$ and vary the $b$ payoff. We have 50 % of each type of agent. We calculate the percentage

**Table 7** Percent of agents that coordinate on their preferred strategy, (X for type E and Y for type F), as a function of the payoff b

| b | m = 0 | | | | m = 1 | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean (%) | SD (%) | Time | Similarity | Mean (%) | SD (%) | Time | Similarity |
| −1 | 49.8 | 4.8 | 2.6 | 49.5 | 98.8 | 5.3 | 29.6 | 98.3 |
| −2 | 40.6 | 5.0 | 2.7 | 49.6 | 96.7 | 8.3 | 36.7 | 95.4 |
| −3 | 33.7 | 5.1 | 2.7 | 49.4 | 89.4 | 13.0 | 45.9 | 85.6 |
| −4 | 27.8 | 5.2 | 2.8 | 49.5 | 88.7 | 13.1 | 47.4 | 84.6 |
| −5 | 22.5 | 5.0 | 2.7 | 49.4 | 85.1 | 14.9 | 51.8 | 80.6 |
| −6 | 19.1 | 4.7 | 2.8 | 49.5 | 81.1 | 18.2 | 49.2 | 78.3 |

Other payoffs: a = 2.0, c = 0.0, d = 1.0. No movement, m = 0 versus movement, m = 1. 12 × 12 lattice. Average number of time periods to reach equilibrium, *similarity* at equilibrium, and SD also reported

of agents that coordinate on their preferred Nash equilibrium (the Nash equilibrium that yields the highest payoff for the agent's type—strategy X for type E and strategy Y for type F) as well as *similarity* of neighbors (the percentage of neighbors that are the same type). The results are reported in Table 7.

Interestingly, when agents are allowed to move, they are able to coordinate on their preferred Nash equilibrium in a very high percentage of runs even when the cost of non-coordination is very great. This is particularly striking when compared to Table 1. In the homogeneous agents case reported in Table 1, when $b = -6$ less than 1 % of agents were able to coordinate on the Pareto dominant Nash equilibrium with movement. Here, over 80 % of agents are able to coordinate on their preferred Nash equilibrium. As one can see from the table they do so by strongly sorting by type. A very large majority of agents sort themselves into groups by type and play the preferred strategy for their type. A typical initial configuration and the resulting equilibrium with movement is shown in Figs. 1 and 2. As seen in the figure, the agents typically sort exclusively (or nearly so) by type and each group plays its preferred strategy. When the *b* payoff is sufficiently large, a configuration like this occurs nearly 100 % of the time.

These results are interesting in two main respects: First, agents are better able to find a spatial region where their favored equilibrium is being played because the diversity of types in the population allows for each strategy (X and Y) to be present. With high probability there is some location on the lattice where an agent's preferred strategy is being played. Once the agent finds a location like this through random movement, the agent moves there, that region grows and further locks in that particular equilibrium locally. The area near this region then becomes even more likely to attract additional agents of this type. If we juxtapose this result with those of the base results section, again we see how movement allows for the attainment of the Pareto dominant Nash equilibrium in the main results of the paper. If the Pareto dominant strategy can survive long enough, movement allows best response to eventually find the region where this strategy is being played. So, as long as agents do not converge too quickly to the risk dominant Nash equilibrium, they are able to find a subset of agents playing the Pareto dominant Nash equilibrium and add to the region of space where this equilibrium

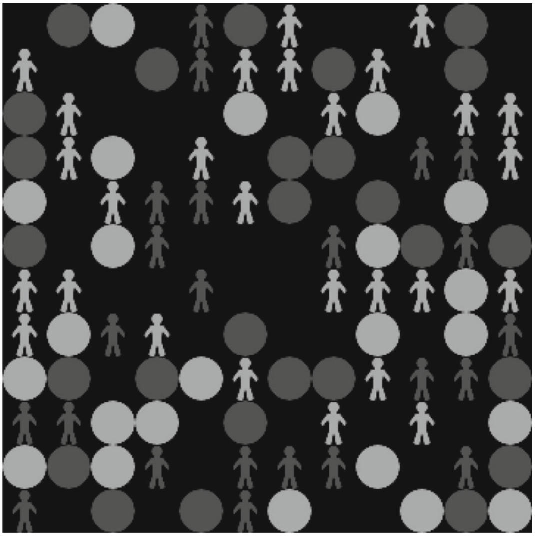

**Fig. 1** Initial random configuration. *Color* represents strategy played. "People" are type E agents, *circles* are type F agents

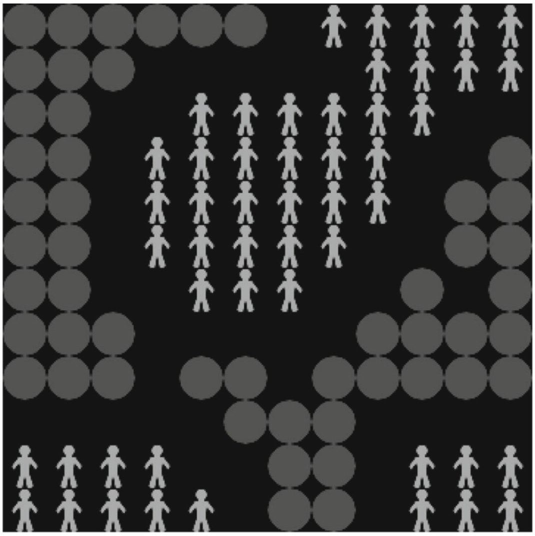

**Fig. 2** Representative equilibrium configuration. *Color* represents strategy played. "People" are type E agents, *circles* are type F agents

occurs. Because this region grows it becomes even easier for other agents of this type to find this region. Thus positive feedbacks lead to the Pareto dominant Nash equilibrium when movement is allowed as long as this strategy can survive initially. Second, here we see an example of the benefits of diversity. Diverse agents allow strategies to survive longer in the population and provide for better opportunities for agents to reach preferred outcomes. With more homogeneous populations, it is easier for strategies to quickly die out and suboptimal outcomes may result.

## 5 Conclusion

The results we report in this paper are important in four main respects: First, we extend the results of movement in prisoner's dilemma games to coordination games and show that the effects of movement are beneficial here as well. Second, we extend the results on equilibrium selection in coordination games. When agents are allowed to move, the attainment of a Pareto dominant Nash equilibrium becomes much more likely. Third, introducing heterogeneous agents into the model can greatly improve coordination game outcomes when agents are allowed to move. This occurs primarily because having a diversity of agent types in the model allows long run beneficial strategies to survive longer in the initial population of agents thereby increasing the probability that an agent can find partners playing her preferred strategy. Without agent diversity this preferred strategy sometimes dies out before agents are able to coordinate on it. Fourth combining our results with the results of Barr and Tassier (2010) on prisoner's dilemma games suggests that agent movement can have very large effects that allow for more cooperative behavior. Perhaps most importantly, the benefits of movement come with very simple strategies. Agents do not need to recognize another agent's type or some other characteristic; they do not need sophisticated strategies with a memory to create reputations or punishments. The agents simply need to be able to leave a location when a better opportunity arrives at a randomly chosen location elsewhere. Yet, even with this simplicity, the ability to move leads to powerful and beneficial effects.

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## Appendix: Additional Results

In addition to the standard model above, we consider two models as robustness tests: random movement and interaction on a torus.[5] First, we consider a random movement model similar to Sicardi et al. (2008) who incorporates random movement into several spatial games such as prisoner's dilemma, snow drift, and stag hunt. In our model agents behave as described above except, when given an opportunity to move, they do so regardless of whether the resulting average payoffs at the new location are better or worse than at her current location. These results are presented in Table 8.

In the table the no movement results are nearly identical to those of Table 1. (This is expected as this is simply another set of random draws of the same no-movement model presented in Sect. 3). However, when random movement occurs, we see much lower rates of Pareto dominant cooperation for values of $b$ where $Y$ is the risk dominant strategy, $b \leq -1$. These results echo those of Helbing and Yu (2009) in that agent movement leads to better outcomes when agents move purposely to locations yielding better outcomes but not otherwise.

5 We thank an anonymous reviewer for suggesting these two experiments.

**Table 8** Random movement results: percent of agents that coordinate on the Pareto dominant strategy, X, as a function of the payoff b

| b | m = 0 | | | m = 1 | | |
|---|---|---|---|---|---|---|
| | Mean (%) | SD (%) | Time | Mean (%) | SD (%) | Time |
| 0 | 98.3 | 3.4 | 1.0 | 100.0 | 0.0 | 4.1 |
| −0.5 | 80.2 | 13.4 | 1.7 | 99.6 | 6.2 | 14.0 |
| −1 | 49.8 | 12.8 | 2.6 | 0.0 | 0.0 | 8.2 |
| −1.1 | 23.9 | 14.4 | 4.3 | 0.0 | 0.0 | 8.0 |
| −1.5 | 24.5 | 14.6 | 4.3 | 0.0 | 0.0 | 6.5 |
| −2 | 8.8 | 8.7 | 4.4 | 0.0 | 0.0 | 3.8 |
| −2.5 | 2.9 | 4.9 | 3.9 | 0.0 | 0.0 | 3.0 |
| −3 | 1.7 | 3.4 | 3.5 | 0.0 | 0.0 | 2.5 |
| −3.5 | 0.4 | 1.3 | 2.7 | 0.0 | 0.0 | 2.4 |
| −4 | 0.4 | 1.3 | 2.6 | 0.0 | 0.0 | 2.3 |
| −4.5 | 0.3 | 1.2 | 2.5 | 0.0 | 0.0 | 2.3 |
| −5 | 0.3 | 1.0 | 2.4 | 0.0 | 0.0 | 2.0 |
| −5.5 | 0.2 | 0.6 | 2.1 | 0.0 | 0.0 | 2.0 |
| −6 | 0.2 | 0.6 | 2.1 | 0.0 | 0.0 | 2.0 |

Other payoffs: a = 2.0, c = 0.0, d = 1.0. No movement, m = 0 versus random movement, m = 1. $12 \times 12$ lattice, $N = 100$ agents. Also reported are the average number of time periods taken to reach equilibrium and the SD

**Table 9** Torus model: percent of agents that coordinate on the Pareto dominant strategy, X, as a function of the payoff b

| b | m = 0 | | | m = 1 | | |
|---|---|---|---|---|---|---|
| | Mean (%) | SD (%) | Time | Mean (%) | SD (%) | Time |
| 0 | 99.9 | 1.0 | 0.9 | 100.0 | 0.0 | 2.9 |
| −0.5 | 86.2 | 13.5 | 1.6 | 100.0 | 0.0 | 4.3 |
| −1 | 49.8 | 14.2 | 2.8 | 100.0 | 1.1 | 8.2 |
| −1.1 | 19.9 | 16.5 | 5.1 | 100.0 | 1.2 | 8.4 |
| −1.5 | 19.8 | 16.4 | 5.1 | 99.1 | 8.9 | 14.1 |
| −2 | 2.9 | 5.8 | 4.4 | 62.2 | 42.6 | 62.8 |
| −2.5 | 0.2 | 1.3 | 3.2 | 38.8 | 42.6 | 63.2 |
| −3 | 0.1 | 1.1 | 2.9 | 5.5 | 15.2 | 25.8 |
| −3.5 | 0.0 | 0.3 | 2.3 | 2.9 | 9.5 | 15.7 |
| −4 | 0.0 | 0.3 | 2.2 | 1.2 | 4.7 | 7.1 |
| −4.5 | 0.0 | 0.2 | 2.2 | 0.8 | 4.1 | 8.1 |
| −5 | 0.0 | 0.1 | 2.1 | 0.3 | 1.9 | 4.4 |
| −5.5 | 0.0 | 0.1 | 1.9 | 0.0 | 0.7 | 2.6 |
| −6 | 0.0 | 0.1 | 2.0 | 0.0 | 0.4 | 2.5 |

Other payoffs: a = 2.0, c = 0.0, d = 1.0. No movement, m = 0 versus movement, m = 1. $12 \times 12$ lattice, $N = 100$ agents. Also reported are the average number of time periods taken to reach equilibrium and the SD

Second, to check for border or edge effects we change our two-dimensional lattice to a torus where edges "wrap around" to the opposite edge. These results are presented in Table 9. As reported in the table, the results are similar to those of Table 1, but with small decreases in the incidence of Pareto dominant coordination for both the no-movement and movement experiments. This likely occurs because edges limit the number of neighbors and make it slightly easier for some agents to achieve Pareto dominant coordination. Overall though, the differences between the base model and the torus model are minimal and the primary result that movement enhances Pareto dominant coordination holds in both models.

## References

Aktipis, C. A. (2004). Know when to walk away: Contingent movement and the evolution of cooperation. *Journal of Theoretical Biology*, *231*(2), 249–260.

Ashlock, D., Smucker, M. D., Stanley, E. A., & Tesfatsion, L. (1996). Preferential partner selection in an evolutionary study of the prisoner's dilemma. *BioSystems*, *37*, 99–125.

Axelrod, R. (1984). *The evolution of cooperation*. New York: Basic Books.

Barr, J., & Tassier, T. (2010). Endogenous neighborhood selection and the attainment of cooperation in a spatial prisoner's dilemma game. *Computational Economics*, *35*, 211–234.

Bednar, J., & Page, S. E. (2007). Can game(s) theory explain culture? The emergence of cultural behavior within multiple games. *Rationality and Society*, *19*, 65–97.

Biely, C., Dragostis, K., & Thurner, S. (2007). The prisoner's dilemma on co-evolving networks under perfect rationality. *Physica D*, *228*, 40–48.

Ellison, G. (1993). Learning, local interaction, and coordination. *Econometrica*, *61*, 1047–1071.

Fort, H. (2008). A minimal model for the evolution of cooperation through evolving heterogeneous games. *Europhysics Letters*, *81*, 48008.

Hanaki, N., Peterhansl, A., Dodds, P. S., & Watts, D. J. (2007). Cooperation in evolving social networks. *Management Science*, *53*, 1036–1050.

Harsanyi, J. C., & Selten, R. (1988). *A general theory of equilibrium selection in games*. Cambridge, MA: MIT Press.

Helbing, D., & Yu, W. (2009). The outbreak of cooperation among success-driven individuals under noisy conditions. *Proceedings of the National Academy of Sciences*, *106*, 3680–3685.

Kandori, M., Mailath, G. J., & Rob, R. (1993). Learning, mutation, and long run equilibria in games. *Econometrica*, *61*, 29–56.

Morris, S. (2000). Contagion. *The Review of Economics Studies*, *67*, 57–78.

Nowak, M. A., & May, R. M. (1992). Evolutionary games and spatial chaos. *Nature*, *359*, 826–829.

Santos, F. C., Pacheco, J. M., & Lenaerts, T. (2006). Cooperation prevails when individuals adjust their social ties. *PLoS Computational Biology*, *2*(10), e140.

Sicardi, E. A., Fort, H., Vainstein, M. H., & Arenzon, J. J. (2008). Random mobility and spatial structure often enhance cooperation. *Journal of Theoretical Biology*, *256*, 240–246.

Van Segbroeck, S., Santos, F. C., Nowe, A., Pacheco, J. M., & Lenaerts, T. (2008). The evolution of prompt reaction to adverse ties. *BMC Evolutionary Biology*, *8*, 287.

Watts, D. J. (1999). *Small worlds: The dynamics of networks between order and randomness*. Princeton, NJ: Princeton University Press.

Young, P. (1993). The evolution of conventions. *Econometrica*, *61*, 57–84.